CBCS SCHEME

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17CS36

Third Semester B.E. Degree Examination, July/August 2021
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions.***

1 a. Write all the logical connectives with truth table. **(06 Marks)**

b. Prove that for any proposition p, q, r the compound proposition $[(p \vee q) \rightarrow r] \Leftrightarrow [\neg r \rightarrow \neg(p \vee q)]$ is logically equivalent. **(08 Marks)**

c. Prove that for any proposition p, q, r the compound proposition $\{p \rightarrow (q \rightarrow r)\} \rightarrow \{(p \rightarrow q) \rightarrow (p \rightarrow r)\}$ is tautology. **(06 Marks)**

2 a. Prove the logical equivalences using laws of logic

i) $[(p \vee q) \wedge (p \vee \neg q)] \vee q \Leftrightarrow p \vee q$

ii) $(p \rightarrow q) \wedge [\neg q \wedge (r \vee \neg q)] \Leftrightarrow \neg(q \vee p)$ **(08 Marks)**

b. Test the validity of the following argument

If I study, I will not fail in the examination

If I don't watch TV in the evening, I will study

I failed in the examination

$\therefore$ I must have watched TV in the evenings **(06 Marks)**

c. Establish the validity of the following argument

$\forall x, \{p(x) \vee q(x)\}$

$\forall x, \{\{\neg p(x) \wedge q(x)\} \rightarrow r(x)\}$

$\therefore \forall x, \{\neg r(x) \rightarrow p(x)\}$ **(06 Marks)**

3 a. Prove that mathematical induction that

$1^2 + 3^2 + 5^2 + \ldots\ldots\ldots\ldots + (2n-1)^2 = \frac{1}{3} n(2n-1)(2n+1)$ **(06 Marks)**

b. For the Fibonacci sequence $F_0, F_1, F_2 \ldots\ldots\ldots$ Prove that

$$F_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right].$$ **(08 Marks)**

c. Find the coefficients of

i) $x^9 y^3$ in the expansion of $(2x-3y)^{12}$

ii) x^{12} in the expansion of $x^3(1-2x)^{10}$. **(06 Marks)**

4 a. In how many ways can 10 identical pencils be distributed among 5 children in the following cases?

i) There are no restrictions

ii) Each child gets atleast one pencil

iii) The youngest child gets atleast two pencils. **(06 Marks)**

b. Prove the following identities :

i) $C(n, r-1) + C(n, r) \equiv C(n+1, r)$

ii) $C(m, 2) + C(n, 2) \equiv C(m+n, 2) - mn$ **(08 Marks)**

c. In how many ways one can distribute eight identical balls into four distinct containers so that
   i) No container is left empty
   ii) The fourth container gets an odd number of balls. **(06 Marks)**

5 a. Consider the function f and g defined by $f(x) = x^3$ and $g(x) = x^2 + 1$, $\forall x \in R$. Find gof, fog, f^2 and g^2. **(06 Marks)**

b. Let A = {1, 2, 3, 4, 6, 12}. On A define the relation R by aRb if and only if a divides b. Prove R is partial order on A. Draw the Hasse diagram for this relation. **(07 Marks)**

c. Let A = {1 2 3 4} and f and g be functions from A to A given by f = {(1, 4) (2, 1), (3, 2), (4, 3)}, g = {(1, 2), (2, 3), (3, 4), (4, 1)}.Prove that f and g are inverse of each other. **(07 Marks)**

6 a. Define an equivalence relation with example. **(08 Marks)**

b. Draw the Hasse diagram representing the positive divisors of 36. **(06 Marks)**

c. Let $f : R \to R$ be defined by

$$f(x) = \begin{cases} 3x - 5 & \text{for } x > 0 \\ -3x + 1 & \text{for } x \le 0 \end{cases}$$

   i) Determine : $f(0)$, $f(-1)$, $f(5/3)$, $f(-5/3)$
   ii) Find $f^{-1}(0)$, $f^{-1}(1)$, $f^{-1}(3)$, $f^{-1}(6)$ **(06 Marks)**

7 a. Out of 30 students in hostel, 15 study History, 8 study Economics and 6 study Geography. It is known that 3 students study all these subjects. Show that 7 or more students study none of these subjects. **(07 Marks)**

b. Find the rook polynomial for the 3×3 board using expansion formula. **(07 Marks)**

c. Solve the recurrence relation $a_n + a_{n-1} - 6a_{n-2} = 0 \quad n \geq 2$ given $a_0 = -1$ $a_1 = 8$. **(06 Marks)**

8 a. An apple, a banana, a mango and an orange are to be distributed among 4 boys B_1, B_2, B_3, B_4. The boys B_1 and B_2 do not wish to have an apple. The boy B_3 does not want banana or mango, B_4 refuses orange. In how many ways the distribution can be made so that no boy is displeased. **(08 Marks)**

b. How many permutations of 1 2 3 4 5 6 7 8 are not derangements? **(05 Marks)**

c. The number of virus affected files in a system is 1000 (to start with) and this increases 250%, every two hours. Use recurrence relation to determine the number of virus affected files in the system after one day. **(07 Marks)**

9 a. Define : i) Graph ii) Simple graph iii) Complete graph iv) Order of graph v) Size of graph vi) Bipertite graph vii) General graph. **(07 Marks)**

b. Show that the following two graphs are isomorphic (Fig Q9(b))

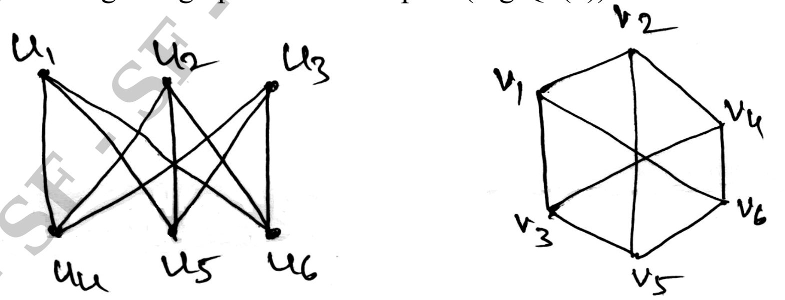

Fig Q9(b) **(06 Marks)**

c. Find the prefix codes for the letters B, E, I, K, L, T, P, S, if the coding scheme is as shown in Fig Q9(c).

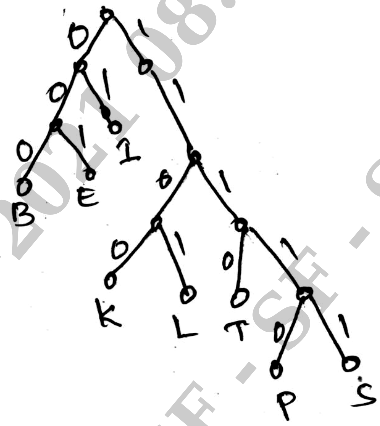

Fig Q9(c)

1) Find the codes for the words PIPE and BEST
2) Decode the string i) 000011100001 ii) 11111111101101011110 **(07 Marks)**

10 a. Obtain an optimal prefix code for the message LETTER RECEIVED. Indicate the code. **(08 Marks)**

b. Apply the merge sort to following list of elements.
{-1, 0, 2, -2, 3, 6, -3, 5, 1, 4}. **(06 Marks)**

c. Let $T_1 = (V_1\ E_1)$ and $T_2 = (V_2\ E_2)$ be two trees. If $|E_1| = 19$ and $|V_2| = 3|V_1|$ determine $|V_1|$, $|V_2|$ & $|E^2|$. **(06 Marks)**

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